Evaluate the following limits. SCORE: _____ / 11 PTS

Your answer should be a number, ∞, −∞ or DNE (only if the first three answers do not apply).

[a] $\lim\limits_{x\to 0} \dfrac{x\sin x}{e^{3x} - 3e^x + 2}$ $\quad \dfrac{0\cdot 0}{1-3+2} \to \dfrac{0}{0}$

$= \boxed{\lim\limits_{x\to 0} \dfrac{\sin x + x\cos x}{3e^{3x} - 3e^x}}$ 2 $\dfrac{0+0}{3-3} \to \dfrac{0}{0}$

$= \boxed{\lim\limits_{x\to 0} \dfrac{\cos x + \cos x - x\sin x}{9e^{3x} - 3e^x}}$ 2

$= \dfrac{1+1-0}{9-3} = \dfrac{2}{6} = \boxed{\dfrac{1}{3}}$ 1

[b] $\lim\limits_{x\to 0^+} (1-\sin x)^{\frac{1}{x}}$

$= \lim\limits_{x\to 0^+} e^{\ln(1-\sin x)^{\frac{1}{x}}}$

$= e^{\lim\limits_{x\to 0^+} \frac{\ln(1-\sin x)}{x}}$

$= \boxed{e^{-1}}$ $1\frac{1}{2}$

$\boxed{\lim\limits_{x\to 0^+} \dfrac{\ln(1-\sin x)}{x}}$ 2 $\dfrac{0}{0}$

$= \boxed{\lim\limits_{x\to 0^+} \dfrac{\frac{1}{1-\sin x}\cdot -\cos x}{1}}$ $1\frac{1}{2}$

$= \dfrac{1}{1-0}\cdot -1 = \boxed{-1}$ 1

Using complete sentences and proper mathematical notation, state the **formal definition** of "local maximum". **SCORE: _____ / 3 PTS**

f HAS A LOCAL MAXIMUM AT C GRADED BY ME

IF $f(x) \le f(c)$ FOR ALL X IN AN OPEN INTERVAL AROUND C

Find the absolute extrema of $f(x) = 3x^{\frac{2}{3}}(x-10)$ on the interval $[-1, 8]$. **SCORE: _____ / 7 PTS**

$f'(x) = 2x^{-\frac{1}{3}}(x-10) + 3x^{\frac{2}{3}}$

$= x^{-\frac{1}{3}}(2x - 20 + 3x)$

$= \boxed{x^{-\frac{1}{3}}(5x-20)}$ (1)

$\boxed{f' \text{ DNE @ } x=0}$ (1)

$\boxed{f' = 0 \text{ @ } x=4}$ (1)

x	$f(x)$
-1	$3(1)(-9) = \underline{-27}$ $(\frac{1}{2})$
0	$\underline{0}$ $(\frac{1}{2})$
4	$3 \cdot 4^{\frac{2}{3}}(-6) = -18\sqrt[3]{16} = \underline{-36\sqrt[3]{2}}$ $(\frac{1}{2})$
8	$3(4)(-2) = \underline{-24}$ $(\frac{1}{2})$

ABSOLUTE $\underline{\text{MAX @ } x=0}$ (1)

$\underline{\text{MIN @ } x=4}$ (1)

Consider the following three cases: **SCORE: _____ / 9 PTS**

Case 1: $f(x) = (x-1)^{\frac{2}{3}}$ on the interval $[-7, 9]$ CONT, NOT DIFF $f' = \frac{2}{3}(x-1)^{-\frac{1}{3}}$ DNE @ $x=1$

Case 2: $g(x) = \frac{4}{4-x}$ on the interval $[0, 3]$ CONT, DIFF $g' = -4(4-x)^{-2}(-1) = \frac{4}{(4-x)^2}$

Case 3: $h(x) = \frac{6}{x^2-1}$ on the interval $[-2, 2]$ DISCONT @ $X = \pm 1$

[a] Does Rolle's Theorem apply to any of the three cases ? **<u>If no, write "N/A" (no other work required).</u>** If yes, list all cases to which it applies, and list all the conditions of Rolle's Theorem which are satisfied.

$\boxed{\text{N/A}}$ 1

[b] Does the Extreme Value Theorem apply to any of the three cases ? **<u>If no, write "N/A" (no other work required).</u>** If yes, list all cases to which it applies, and list all the conditions of the Extreme Value Theorem which are satisfied.

CASES $\boxed{1, 2}$: 1

FUNCTIONS ARE $\boxed{\text{CONTINUOUS}}$ ON $\boxed{\text{CLOSED + BOUNDED INTERVALS}}$

1 $\frac{1}{2}$

[c] Does the Mean Value Theorem apply to any of the three cases ? **<u>If no, write "N/A" (no other work required).</u>** If yes, list all cases to which it applies, and for each of those cases, list all the conditions of the Mean Value Theorem which are satisfied, and find the value of c guaranteed by the Mean Value Theorem.

CASE $\boxed{2}$ 1 g IS $\boxed{\text{CONT + DIFF}}$ $1\frac{1}{2}$

$g'(c) = \frac{g(3)-g(0)}{3-0} = \frac{4-1}{3} = 1$

2 $\boxed{\frac{4}{(4-c)^2} = 1} \longrightarrow 4 = (4-c)^2 \longrightarrow 4-c = \pm 2 \longrightarrow c = 2, 6$ 1 $\boxed{c=2} \in (0,3)$